GLMP of deforestation evolution in municipalities of the Amazon region

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This document describes in depth the Granular Linguistic Model of Phenomena (GLMP) for the work related to deforestation in the Amazon region. Section 1 presents the GLMP with all the new contributions developed in this work. Section 2 presents the same GLMP without the new contributions developed in this work.

1 GLMP for Advance Reports

1.1 GLMP for analyzing the level of deforestation

$\mathbf{1PM}_d$ = (U_d, y_d, g_d, T_d), where d stands for *deforestation* and:

U_d = (year, percentage of deforestation, percentage of cloudiness)

y_d = (A_d, W_{Ad}, R_{Ad}, B_d, W_{Bd}) where:

A_d = (*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*)

W_{Ad} = ($w_{Ad_{zero}}$, $w_{Ad_{near\ to\ zero}}$, $w_{Ad_{very\ low}}$, $w_{Ad_{low}}$, $w_{Ad_{moderate\ low}}$, $w_{Ad_{moderate}}$, $w_{Ad_{moderate\ high}}$, $w_{Ad_{high}}$, $w_{Ad_{very\ high}}$), with $w_{Ad_i} \in$ [0,1]

R_{Ad} = ($r_{Ad_{zero}} = 1$, $r_{Ad_{near\ to\ zero}} = 1$, $r_{Ad_{very\ low}} = 1$, $r_{Ad_{low}} = 1$, $r_{Ad_{moderate\ low}} = 1$, $r_{Ad_{moderate}} = 1$, $r_{Ad_{moderate\ high}} = 1$, $r_{Ad_{high}} = 1$, $r_{Ad_{very\ high}} = 1$)

B_d = (*low*, *moderate*, *high*)

W_{Bd} = ($w_{Bd_{low}}, w_{Bd_{moderate}}, w_{Bd_{high}}$), with $w_{Bd_i} \in$ [0,1]

g_d :

g_{W_A}: SFP(*zero*=$triangle_{MF}$(0, 0, 0.001), *near to zero*=$triangle_{MF}$(0, 0.001, 0.1), *very low*=$triangle_{MF}$(0.001, 0.1, 15), *low*=$triangle_{MF}$(0.1, 15, 30), *moderate low*=$triangle_{MF}$(15, 30, 45), *moderate*=$triangle_{MF}$(30, 45, 55), *moderate high*=$trapezoid_{MF}$(45, 55, 60, 70), *high*=$trapezoid_{MF}$(60, 70, 80, 85), *very high*=$trapezoid_{MF}$(80, 85, 100, 100))

g_{W_B}: SFP(*low*=$trapezoid_{MF}$(3, 10, 100, 100), *moderate*=$triangle_{MF}$(0, 3, 10), *high*=$trapezoid_{MF}$(0, 0, 0, 3))

NOTE: SFP stands for Strong Fuzzy Partition. $triangle_{MF}$ is a membership function with triangular shape and three parameters (a, b, c) determining the vertices of the triangle. $trapezoid_{MF}$ is a membership function with trapezoidal shape and four parameters (a, b, c, d) determining the support (a-d) and the core (b-c) of the trapezoid.

T_d : "The deforestation in {*year*} was {*zero* | *near to zero* | *very low* | *low* | *moderate low* | *moderate* | *moderate high* | *high* | *very high*}. Because of the cloudiness, the reliability of this information is {*low* | *moderate* | *high*}".

2PM$_{Ed}$ = (U_{Ed}, y_{Ed}, g_{Ed}, T_{Ed}), where Ed means that this is an *Emphatic* Perception Mapping related to *deforestation*:

U_{Ed} = (CP_{d2000}, CP_{d2001}, CP_{d2002}, CP_{d2003}, CP_{d2004}, CP_{d2005}, CP_{d2006}, CP_{d2007}, CP_{d2008}, CP_{d2009}, CP_{d2010}, CP_{d2011}, CP_{d2012}, CP_{d2013}, CP_{d2014})

y_{Ed} = (A_{Ed}, W_{AEd}, R_{AEd}, B_{Ed}, W_{BEd}) where:

A_{Ed} = (a_{Ed_1}, a_{Ed_2}, ..., $a_{Ed_{32767}}$). Note that the linguistic expressions in A_{Ed} are combinations of n elements taken m at a time being n=15 (the number of elements in U_{Ed}), m={1, 2, ..., 15}, where a_{Ed_1} is the first combination that contains the first element in U_{Ed}: a_{Ed_1}=(CP_{d2000}).

W_{AEd} = (w_{AEd_1}, w_{AEd_2}, ..., $w_{AEd_{32767}}$), with $w_{AEd_i} \in$ [0,1]

R_{AEd} = ($r_{AEd_1} = 1$, $r_{AEd_2} = 1$, ..., $r_{AEd_{32767}} = 1$)

B_{Ed} = (*low*, *moderate*, *high*)

W_{BEd} = ($w_{BEd_{low}} = 0, w_{BEd_{moderate}} = 0, w_{BEd_{high}} = 1$)

g_{Ed} :

g_{W_A}: We look for combinations of elements in U_{Ed}, with the greatest possible number of elements having $w_{low} > 0.5$. We set the validity degree of this combination to the maximum value ($w_{a_{Ed}} = 1$).

T_{Ed} : "Because of the cloudiness, the reliability of our assessment about the level of deforestation in {$year_1$, ... and $year_m$} is lower than the one we could obtain with sunny weather".

2PM$_{Id}$ = (U_{Id}, y_{Id}, g_{Id}, T_{Id}), where Id means that this is an *Integrative* Perception Mapping related to *deforestation*:

U_{Id} = (CP_{d2000}, CP_{d2001}, CP_{d2002}, CP_{d2003}, CP_{d2004}, CP_{d2005}, CP_{d2006}, CP_{d2007}, CP_{d2008}, CP_{d2009}, CP_{d2010}, CP_{d2011}, CP_{d2012}, CP_{d2013}, CP_{d2014})

y_{Id} = (A_{Id}, W_{AId}, R_{AId}, B_{Id}, W_{BId}) where:

A_{Id} = ((Q_{T_1}, Q_{L_1}), ..., (Q_{T_1}, Q_{L_9}), (Q_{T_2}, Q_{L_1}), ..., (Q_{T_2}, Q_{L_9}), ..., (Q_{T_6}, Q_{L_1}), ..., (Q_{T_6}, Q_{L_9})), being Q_T=(*one*, *two*, *three*, *several*, *most*, *all*) and Q_L=(*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*)

W_{AId} = ($w_{AId_{11}}$ = (*one*, *zero*), ..., $w_{AId_{19}}$ = (*one*, *very high*), $w_{AId_{21}}$ = (*two*, *zero*), ..., $w_{AId_{29}}$ = (*two*, *very high*), ..., $w_{AId_{61}}$ = (*all*, *zero*), ..., $w_{AId_{69}}$ = (*all*, *very high*)), with $w_{AId_{ij}} \in$ [0,1]

R_{AId} = ($w_{AId_{11}} = 1$, ..., $w_{AId_{19}} = 1$, $w_{AId_{21}} = 1$, ..., $w_{AId_{29}} = 1$, ..., $w_{AId_{61}} = 1$, ..., $w_{AId_{69}} = 1$)

B_{Id} = (*low*, *moderate*, *high*)

W_{BId} = $(w_{BId_{low}} = 0, w_{BId_{moderate}} = 0, w_{BId_{high}} = 1)$

g_{Id} :

g_{W_A}: For each level of deforestation determined by the index j, i.e., A_d= (a_{d_1}, a_{d_2}, ..., a_{d_j}), we calculate the percentage of years contained at each $\alpha-$level ($N_{\alpha j}$), with $\alpha \in$ A= (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1), as follows:

$$N_{\alpha j} = \frac{1}{K} \sum_{k=1}^{K} F_\alpha(w_{d_j}[k]) \tag{1}$$

where $F_\alpha(z)$ is defined by (2), and $w_{d_j}[k]$ is the validity degree of CP_d, which is calculated for each year (k) being K the total number of evaluated years of each municipality.

$$F_\alpha(z) = \begin{cases} 1 & \text{if} \quad z \geq \alpha \\ 0 & \text{if} \quad z < \alpha \end{cases} \tag{2}$$

Then, we calculate the membership degree of each $N_{\alpha j}$ to each element in Q_T, e.g., $\mu_{Q_{T1}}(N_{\alpha j}) = one(N_{\alpha j})$. The shapes of these linguistic labels are determined by the total number of years K. The related SFP is as follows: (*one*=$trapezoid_{MF}$(0, 0, 0.1, 0.2), *two*=$triangle_{MF}$(0.1, 0.2, 0.3), *three*=$triangle_{MF}$(0.2, 0.3, 0.4), *several*=$triangle_{MF}$(0.3, 0.4, 0.6), *most*=$trapezoid_{MF}$(0.4, 0.6, 0.99, 1), *all*=$triangle_{MF}$(0.99, 1, 1)).

The last step is to calculate the average value of the membership degrees obtained for each $\alpha-$level using (3). The cardinality of the set A ($|\text{A}| = 10$) represents the resolution degree.

$$w_{Id_{ij}} = \frac{1}{|\text{A}|} \sum_{\forall \alpha \in \text{A}} \mu_{Q_{T_i}}(N_{\alpha j}) \tag{3}$$

This final value contains the relevant information about the amount of years with (*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*) deforestation level.

T_{Id} :

If the combination of linguistic expressions in CP_{Id} with the greatest validity degree is $A_{Id_{61}}$ = (*all*, *zero*), then the generated sentence will be: "Results show that in the last fifteen years there have not been deforestation".

Otherwise, we apply the following linguistic template: "Results show that {*the greatest* Q_T} the deforestation level was {*the greatest* Q_L}, and besides, {*the second greatest* Q_T} the deforestation level was {*the second greatest* Q_L}".

1.2 GLMP for analyzing the increment in the level of deforestation

1PM$_i$ = (U_i, y_i, g_i, T_i), where i stands for *increment* and:

U_i = (the year, the increment in the percentage of deforestation)

y_i = (A_i, W_{Ai}, R_{Ai}, B_i, W_{Bi}) where:

A_i = (near to zero, very low, low, moderate, moderate high, high, very high)

W_{Ai} = ($w_{Ai_{near\ to\ zero}}$, $w_{Ai_{very\ low}}$, $w_{Ai_{low}}$, $w_{Ai_{moderate}}$, $w_{Ai_{moderate\ high}}$, $w_{Ai_{high}}$, $w_{Ai_{very\ high}}$), with $w_{Ai_i} \in [0, 1]$

R_{Ai} = ($r_{Ai_{near\ to\ zero}} = 1$, $r_{Ai_{very\ low}} = 1$, $r_{Ai_{low}} = 1$, $r_{Ai_{moderate}} = 1$, $r_{Ai_{moderate\ high}} = 1$, $r_{Ai_{high}} = 1$, $r_{Ai_{very\ high}} = 1$)

B_i = (*low*, *moderate*, *high*)

W_{Bi} = ($w_{Bi_{low}} = 0, w_{Bi_{moderate}} = 0, w_{Bi_{high}} = 1$)

g_i :

g_{W_A}: SFP(*near to zero*=$triangle_{MF}$(0, 0, 0.1), *very low*=$triangle_{MF}$(0, 0.1, 15), *low*=$triangle_{MF}$(0.1, 15, 30), *moderate*=$triangle_{MF}$(15, 30, 45), *moderate high*=$triangle_{MF}$(30, 45, 60), *high*=$trapezoid_{MF}$(45, 60, 70, 80), *very high*=$trapezoid_{MF}$(70, 80, 100, 100))

T_i : "The increment in the deforestation in {*year*} was {*near to zero* | *very low* | *low* | *moderate* | *moderate high* | *high* | *very high*}".

1PM$_{imax}$ = (U_{imax}, y_{imax}, g_{imax}, T_{imax}), where *imax* means *maximum increment* and:

U_{imax} = (year, deforestation increment)

y_{imax} = (A_{imax}, W_{Aimax}, R_{Aimax}, B_{imax}, W_{Bimax}) where:

A_{imax} = (*yes*, *no*)

W_{Aimax} = ($w_{Aimax_{yes}}$, $w_{Aimax_{no}}$), with $w_{Aimax_i} \in [0, 1]$

$$R_{Aimax} = (r_{Aimax_{yes}} = 1, r_{Aimax_{no}} = 1)$$
$$B_{imax} = (low, moderate, high)$$
$$W_{Bimax} = (w_{Bimax_{low}} = 0, w_{Bimax_{moderate}} = 0, w_{Bimax_{high}} = 1)$$

g_{imax} :

g_{W_A}: For each U_{imax}, we create a new CP_{imax} with $w_{Aimax_{yes}}$=1 and $w_{Aimax_{no}}$=0, in case of local maximum. Otherwise, $w_{Aimax_{yes}}$=0 and $w_{Aimax_{no}}$=1.

T_{imax} : "The increment in the deforestation in {*year*} {*was* | *was not*} a maximum". Notice that we select {*was*} when the validity degree associated to the linguistic expression *yes* is equal to 1 and {*was not*} in the other case.

1PM$_{imin}$ = (U_{imin}, y_{imin}, g_{imin}, T_{imin}), where *imin* means *minimum increment* and:

U_{imin} = (year, deforestation increment)

y_{imin} = (A_{imin}, W_{Aimin}, R_{Aimin}, B_{imin}, W_{Bimin}) where:

$$A_{imin} = (yes, no)$$
$$W_{Aimin} = (w_{Aimin_{yes}}, w_{Aimin_{no}}), \text{ with } w_{Aimin_i} \in [0, 1]$$
$$R_{Aimin} = (r_{Aimin_{yes}} = 1, r_{Aimin_{no}} = 1)$$
$$B_{imin} = (low, moderate, high)$$
$$W_{Bimin} = (w_{Bimin_{low}} = 0, w_{Bimin_{moderate}} = 0, w_{Bimin_{high}} = 1)$$

g_{imin} :

g_{W_A}: For each U_{imin}, we create a new CP_{imin} with $w_{Aimin_{yes}}$=1 and $w_{Aimin_{no}}$=0, in case of local minimum. Otherwise, $w_{Aimin_{yes}}$=0 and $w_{Aimin_{no}}$=1.

T_{imin} : "The increment in the deforestation in {*year*} {*was* | *was not*} a minimum". Notice that we select {*was*} when the validity degree associated to the linguistic expression *yes* is equal to 1 and {*was not*} in the other case.

2PM$_{Eimax}$ = (U_{Eimax}, y_{Eimax}, g_{Eimax}, T_{Eimax}), where *Eimax* means *Emphatic* Perception Mapping related to *maximum increment* and:

U_{Eimax} = ($CP_{imax2000}$, $CP_{imax2001}$, $CP_{imax2002}$, $CP_{imax2003}$, $CP_{imax2004}$, $CP_{imax2005}$, $CP_{imax2006}$, $CP_{imax2007}$, $CP_{imax2008}$, $CP_{imax2009}$, $CP_{imax2010}$, $CP_{imax2011}$, $CP_{imax2012}$, $CP_{imax2013}$, $CP_{imax2014}$)

y_{Eimax} = (A_{Eimax}, W_{AEimax}, R_{AEimax}, B_{Eimax}, W_{BEimax}) where:

A_{Eimax} = (a_{Eimax_1}, a_{Eimax_2}, ..., $a_{Eimax_{32767}}$). Note that the linguistic expressions in A_{Eimax} are combinations of n elements taken m at a time being n=15 (the number of elements in U_{Eimax}), m={1, 2, ..., 15}, where a_{Eimax_1} is the first combination that contains the first element in U_{Eimax}: a_{Eimax_1}=($CP_{imax2000}$).

W_{AEimax} = = (w_{AEimax_1}, w_{AEimax_2}, ..., $w_{AEimax_{32767}}$), with $w_{AEimax_i} \in$ [0,1]

R_{AEimax} = ($r_{AEimax_1} = 1$, $r_{AEimax_2} = 1$, ..., $r_{AEimax_{32767}} = 1$)

B_{Eimax} = (*low*, *moderate*, *high*)

W_{BEimax} = ($w_{BEimax_{low}} = 0, w_{BEimax_{moderate}} = 0, w_{BEimax_{high}} = 1$)

g_{Eimax} :

g_{W_A}: We look for combinations, with the greatest possible number of elements having $w_{Aimax_{yes}} = 1$ and increment $\geq$ average increment. We set the validity degree of this combination to the maximum value ($w_{AEimax} = 1$).

T_{Eimax} :

If the combination with the greatest validity degree contains more than one CP_{imax} ($m > 1$), then we apply the linguistic template: "the relevant maximum values in {$year_1$}, ... and {$year_m$}".

Otherwise, we apply the linguistic template: "the relevant maximum value in {$year_1$}".

2PM$_{Eimin}$ = (U_{Eimin}, y_{Eimin}, g_{Eimin}, T_{Eimin}), where *Eimin* means *Emphatic* Perception Mapping related to *minimum increment* and:

U_{Eimin} = ($CP_{imin2000}$, $CP_{imin2001}$, $CP_{imin2002}$, $CP_{imin2003}$, $CP_{imin2004}$, $CP_{imin2005}$, $CP_{imin2006}$, $CP_{imin2007}$, $CP_{imin2008}$, $CP_{imin2009}$, $CP_{imin2010}$, $CP_{imin2011}$, $CP_{imin2012}$, $CP_{imin2013}$, $CP_{imin2014}$)

y_{Eimin} = (A_{Eimin}, W_{AEimin}, R_{AEimin}, B_{Eimin}, W_{BEimin}) where:

A_{Eimin} = (a_{Eimin_1}, a_{Eimin_2}, ..., $a_{Eimin_{32767}}$). Note that the linguistic expressions in A_{Eimin} are combinations of n elements taken m at a time being n=15 (the number of elements in U_{Eimin}), m={1, 2, ..., 15}, where a_{Eimin_1} is the first combination that contains the first element in U_{Eimin}: a_{Eimin_1}=($CP_{imin2000}$).

W_{AEimin} = = (w_{AEimin_1}, w_{AEimin_2}, ..., $w_{AEimin_{32767}}$), with $w_{AEimin_i} \in$ [0,1]

R_{AEimin} = ($r_{AEimin_1} = 1$, $r_{AEimin_2} = 1$, ..., $r_{AEimin_{32767}} = 1$)

B_{Eimin} = (*low*, *moderate*, *high*)

W_{BEimin} = ($w_{BEimin_{low}} = 0, w_{BEimin_{moderate}} = 0, w_{BEimin_{high}} = 1$)

g_{Eimin} :

g_{W_A}: We look for combinations, with the greatest possible number of elements having $w_{Aimin_{yes}} = 1$ and increment $\leq$ average increment. We set the validity degree of this combination to the maximum value ($w_{AEimin} = 1$).

T_{Eimin} :

If the combination with the least validity degree contains more than one CP_{imin} ($m > 1$), then we apply the linguistic template: "the relevant minimum values in $\{year_1\}$, ... and $\{year_m\}$".

Otherwise, we apply the linguistic template: "the relevant minimum value in $\{year_1\}$".

2PM$_{Ci}$ $= (U_{Ci}, y_{Ci}, g_{Ci}, T_{Ci})$, where Ci means *Comparative* Perception Mapping related to *increment*:

$U_{Ci} = (CP_{i_1}, CP_{i_2})$, being CP_{i_1} the supreme minimum and CP_{i_2} the supreme maximum.

$y_{Ci} = (A_{Ci}, W_{ACi}, R_{ACi}, B_{Ci}, W_{BCi})$ where:

A_{Ci} = (*much lower than*, *lower than*, *similar to*, *greater than*, *much greater than*)

$W_{ACi} = (w_{ACi_{much\ lower\ than}}, w_{ACi_{lower\ than}}, w_{ACi_{similar\ to}}, w_{ACi_{greater\ than}}, w_{ACi_{much\ greater\ than}})$, with $w_{ACi_i} \in [0, 1]$

$R_{ACi} = (r_{ACi_{much\ lower\ than}} = 1,\ r_{ACi_{lower\ than}} = 1,\ r_{ACi_{similar\ to}} = 1, r_{ACi_{greater\ than}} = 1,\ r_{ACi_{much\ greater\ than}} = 1)$

B_{Ci} = (*low*, *moderate*, *high*)

$W_{BCi} = (w_{BCi_{low}} = 0, w_{BCi_{moderate}} = 0, w_{BCi_{high}} = 1)$

g_{Ci} :

g_{W_A}: a set of fuzzy rules which makes a comparison between two different instances of the same CP as follows:

R1: **IF** A_{i_1} is *near to zero* AND A_{i_2} is *near to zero* **THEN** CP_{Ci} is *similar to*

R2: **IF** A_{i_1} is *near to zero* AND A_{i_2} is *very low OR low OR moderate* **THEN** CP_{Ci} is *greater than*

R3: **IF** A_{i_1} is *near to zero* AND A_{i_2} is *moderate high OR high OR very high* **THEN** CP_{Ci} is *much greater than*

R4: **IF** A_{i_1} is *very low* AND A_{i_2} is *near to zero* **THEN** CP_{Ci} is *lower than*

R5: **IF** A_{i_1} is *very low* AND A_{i_2} is *very low* **THEN** CP_{Ci} is *similar to*

R6: **IF** A_{i_1} is *very low* AND A_{i_2} is *low OR moderate* **THEN** CP_{Ci} is *greater than*

R7: **IF** A_{i_1} is *very low* AND A_{i_2} is *moderate high OR high OR very high* **THEN** CP_{Ci} is *much greater than*

R8: **IF** A_{i_1} is *low* AND A_{i_2} is *near to zero OR very low* **THEN** CP_{Ci} is *lower than*

R9: **IF** A_{i_1} is *low* AND A_{i_2} is *low* **THEN** CP_{Ci} is *similar to*

R10: **IF** A_{i_1} is *low* AND A_{i_2} is *moderate OR moderate high* **THEN** CP_{Ci} is *greater than*

R11: **IF** A_{i_1} is *low* AND A_{i_2} is *high OR very high* **THEN** CP_{Ci} is *much greater than*

R12: **IF** A_{i_1} is *moderate* AND A_{i_2} is *near to zero* **THEN** CP_{Ci} is *much lower than*

R13: **IF** A_{i_1} is *moderate* AND A_{i_2} is *very low OR low* **THEN** CP_{Ci} is *lower than*

R14: **IF** A_{i_1} is *moderate* AND A_{i_2} is *moderate* **THEN** CP_{Ci} is *similar to*

R15: **IF** A_{i_1} is *moderate* AND A_{i_2} is *moderate high OR high* **THEN** CP_{Ci} is *greater than*

R16: **IF** A_{i_1} is *moderate* AND A_{i_2} is *very high* **THEN** CP_{Ci} is *much greater than*

R17: **IF** A_{i_1} is *moderate high* AND A_{i_2} is *near to zero OR very low* **THEN** CP_{Ci} is *much lower than*

R18: **IF** A_{i_1} is *moderate high* AND A_{i_2} is *low OR moderate* **THEN** CP_{Ci} is *lower than*

R19: **IF** A_{i_1} is *moderate high* AND A_{i_2} is *moderate high* **THEN** CP_{Ci} is *similar to*

R20: **IF** A_{i_1} is *moderate high* AND A_{i_2} is *high OR very high* **THEN** CP_{Ci} is *greater than*

R21: **IF** A_{i_1} is *high* AND A_{i_2} is *near to zero OR very low* **THEN** CP_{Ci} is *much lower than*

R22: **IF** A_{i_1} is *high* AND A_{i_2} is *low OR moderate OR moderate high* **THEN** CP_{Ci} is *lower than*

R23: **IF** A_{i_1} is *high* AND A_{i_2} is *high* **THEN** CP_{Ci} is *similar to*

R24: **IF** A_{i_1} is *high* AND A_{i_2} is *very high* **THEN** CP_{Ci} is *greater than*

R25: **IF** A_{i_1} is *very high* AND A_{i_2} is *near to zero OR very low OR low* **THEN** CP_{Ci} is *much lower than*

R26: **IF** A_{i_1} is *very high* AND A_{i_2} is *moderate OR moderate high OR high* **THEN** CP_{Ci} is *lower than*

R27: **IF** A_{i_1} is *very high* AND A_{i_2} is *very high* **THEN** CP_{Ci} is *similar to*

T_{Ci} : "The supreme minimum is {*much lower than* | *lower than* | *similar to* | *greater than* | *much greater than*} the supreme maximum".

1.3 GLMP for analyzing the acceleration in the deforestation rate

1PM$_a$ $= (U_a, y_a, g_a, T_a)$, where a stands for *acceleration* and:

U_a = (the previous year, the current year, the percentage of acceleration between these years)

$y_a = (A_a, W_{Aa}, R_{Aa}, B_a, W_{Ba})$ where:

A_a = (*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*)

W_{Aa} = $(w_{Aa_{zero}}, w_{Aa_{near\ to\ zero}}, w_{Aa_{very\ low}}, w_{Aa_{low}}, w_{Aa_{moderate\ low}}, w_{Aa_{moderate}}, w_{Aa_{moderate\ high}}, w_{Aa_{high}}, w_{Aa_{very\ high}})$, with $w_{Aa_i} \in [0,1]$

R_{Aa} = $(r_{Aa_{zero}} = 1,\ r_{Aa_{near\ to\ zero}} = 1,\ r_{Aa_{very\ low}} = 1,\ r_{Aa_{low}} = 1, r_{Aa_{moderate\ low}} = 1, r_{Aa_{moderate}} = 1, r_{Aa_{moderate\ high}} = 1, r_{Aa_{high}} = 1, r_{Aa_{very\ high}} = 1)$

B_a = (*low*, *moderate*, *high*)

W_{Ba} = $(w_{Ba_{low}} = 0, w_{Ba_{moderate}} = 0, w_{Ba_{high}} = 1)$

g_a :

g_{W_A}: SFP(*zero*=$triangle_{MF}(0, 0, 0.001)$, *near to zero*=$triangle_{MF}(0, 0.001, 0.1)$, *very low*=$triangle_{MF}(0.001, 0.1, 15)$, *low*=$triangle_{MF}(0.1, 15, 30)$, *moderate low*=$triangle_{MF}(15, 30, 45)$, *moderate*=$triangle_{MF}(30, 45, 55)$, *moderate high*=$trapezoid_{MF}(45, 55, 60, 70)$, *high*=$trapezoid_{MF}(60, 70, 80, 85)$, *very high*=$trapezoid_{MF}(80, 85, 100, 100)$).

T_a : "The acceleration in the deforestation between years {*previous year-current year*} was {*zero* | *near to zero* | *very low* | *low* | *moderate low* | *moderate* | *moderate high* | *high* | *very high*}".

2PM$_{Samax}$ = (U_{Samax}, y_{Samax}, g_{Samax}, T_{Samax}), where *Samax* means *Superlative* Perception Mapping related to *maximum acceleration* and:

U_{Samax} is a vector with all CP_a for a given municipality.

y_{Samax} = (A_{Samax}, W_{ASamax}, R_{ASamax}, B_{Samax}, W_{BSamax}) where:

A_{Samax} = (*small*, *medium*, *high*)

W_{ASamax} = ($w_{ASamax_{small}}$, $w_{ASamax_{medium}}$, $w_{ASamax_{high}}$), with $w_{ASamax_i} \in [0, 1]$

R_{ASamax} = ($r_{ASamax_{small}} = 1$, $r_{ASamax_{medium}} = 1$, $r_{ASamax_{high}} = 1$)

B_{Samax} = (*low*, *moderate*, *high*)

W_{BSamax} = ($w_{BSamax_{low}} = 0, w_{BSamax_{moderate}} = 0, w_{BSamax_{high}} = 1$)

g_{Samax} :

g_{W_A}: Firstly, we rank the elements in U_{Samax} in descending order by the acceleration linguistic labels. The first element in the list is identified as the one with the most significant acceleration. The comparison between pairs of instances CP_{a_i} and CP_{a_j} is made by the next rule base:

R1: **IF** CP_{a_i} is *near to zero* AND CP_{a_j} is *near to zero* **THEN** CP_{Ch} is *similar to*

R2: **IF** CP_{a_i} is *near to zero* AND CP_{a_j} is *very low OR low OR moderate* **THEN** CP_{Ch} is *greater than*

R3: **IF** CP_{a_i} is *near to zero* AND CP_{a_j} is *moderate high OR high OR very high* **THEN** CP_{Ch} is *much greater than*

R4: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *near to zero* **THEN** CP_{Ch} is *lower than*

R5: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *very low* **THEN** CP_{Ch} is *similar to*

R6: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *low OR moderate* **THEN** CP_{Ch} is *greater than*

R7: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *moderate high OR high OR very high* **THEN** CP_{Ch} is *much greater than*

R8: **IF** CP_{a_i} is *low* AND CP_{a_j} is *near to zero OR very low* **THEN** CP_{Ch} is *lower than*

R9: **IF** CP_{a_i} is *low* AND CP_{a_j} is *low* **THEN** CP_{Ch} is *similar to*

R10: **IF** CP_{a_i} is *low* AND CP_{a_j} is *moderate OR moderate high* **THEN** CP_{Ch} is *greater than*

R11: **IF** CP_{a_i} is *low* AND CP_{a_j} is *high OR very high* **THEN** CP_{Ch} is *much greater than*

R12: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *near to zero* **THEN** CP_{Ch} is *much lower than*

R13: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *very low OR low* **THEN** CP_{Ch} is *lower than*

R14: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *moderate* **THEN** CP_{Ch} is *similar to*

R15: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *moderate high OR high* **THEN** CP_{Ch} is *greater than*

R16: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *very high* **THEN** CP_{Ch} is *much greater than*

R17: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *near to zero OR very low* **THEN** CP_{Ch} is *much lower than*

R18: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *low OR moderate* **THEN** CP_{Ch} is *lower than*

R19: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *moderate high* **THEN** CP_{Ch} is *similar to*

R20: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *high OR very high* **THEN** CP_{Ch} is *greater than*

R21: **IF** CP_{a_i} is *high* AND CP_{a_j} is *near to zero OR very low* **THEN** CP_{Ch} is *much lower than*

R22: **IF** CP_{a_i} is *high* AND CP_{a_j} is *low OR moderate OR moderate high* **THEN** CP_{Ch} is *lower than*

R23: **IF** CP_{a_i} is *high* AND CP_{a_j} is *high* **THEN** CP_{Ch} is *similar to*

R24: **IF** CP_{a_i} is *high* AND CP_{a_j} is *very high* **THEN** CP_{Ch} is *greater than*

R25: **IF** CP_{a_i} is *very high* AND CP_{a_j} is *near to zero OR very low OR low* **THEN** CP_{Ch} is *much lower than*

R26: **IF** CP_{a_i} is *very high* AND CP_{a_j} is *moderate OR moderate high OR high* **THEN** CP_{Ch} is *lower than*

R27: **IF** CP_{a_i} is *very high* AND CP_{a_j} is *very high* **THEN** CP_{Ch} is *similar to*

Then, we compute the difference of the most significant acceleration with respect to the average of the rest of elements. We compute this difference by the next SFP(*small*=$triangle_{MF}$(0, 0, 20), *medium*=$triangle_{MF}$(0, 20, 50), *high*=$trapezoid_{MF}$(20, 50, 100, 100)).

T_{Samax} : "The greatest acceleration took place between years {*previous year*}-{*current year*} by {*small* | *medium* | *high*} difference with respect to the average of the rest of years".

2PM$_{Samin}$ $= (U_{Samin}, y_{Samin}, g_{Samin}, T_{Samin})$, where *Samin* means *Superlative* Perception Mapping related to *minimum acceleration* and:

U_{Samin} is a vector with all CP_a for a given municipality.

$y_{Samin} = (A_{Samin}, W_{ASamin}, R_{ASamin}, B_{Samin}, W_{BSamin})$ where:

$$A_{Samin} = (small, medium, high)$$

$$W_{ASamin} = (w_{ASamin_{small}}, w_{ASamin_{medium}}, w_{ASamin_{high}}), \text{ with } w_{ASamax} \in [0,1]$$

$$R_{ASamin} = (r_{ASamin_{small}} = 1, r_{ASamin_{medium}} = 1, r_{ASamin_{high}} = 1)$$

$$B_{Samin} = (low, moderate, high)$$

$$W_{BSamin} = (w_{BSamin_{low}} = 0, w_{BSamin_{moderate}} = 0, w_{BSamin_{high}} = 1)$$

g_{Samin} :

g_{W_A}: Firstly, we rank the elements in U_{Samin} in descending order by the acceleration linguistic labels. The first element in the list is identified as the one with the lowest acceleration. The comparison between pairs of instances CP_{a_i} and CP_{a_j} is made by the next rule base:

R1: **IF** CP_{a_i} is *near to zero* AND CP_{a_j} is *near to zero* **THEN** CP_{Ch} is *similar to*

R2: **IF** CP_{a_i} is *near to zero* AND CP_{a_j} is *very low OR low OR moderate* **THEN** CP_{Ch} is *greater than*

R3: **IF** CP_{a_i} is *near to zero* AND CP_{a_j} is *moderate high OR high OR very high* **THEN** CP_{Ch} is *much greater than*

R4: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *near to zero* **THEN** CP_{Ch} is *lower than*

R5: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *very low* **THEN** CP_{Ch} is *similar to*

R6: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *low OR moderate* **THEN** CP_{Ch} is *greater than*

R7: **IF** CP_{a_i} is *very low* AND CP_{a_j} is *moderate high OR high OR very high* **THEN** CP_{Ch} is *much greater than*

R8: **IF** CP_{a_i} is *low* AND CP_{a_j} is *near to zero OR very low* **THEN** CP_{Ch} is *lower than*

R9: **IF** CP_{a_i} is *low* AND CP_{a_j} is *low* **THEN** CP_{Ch} is *similar to*

R10: **IF** CP_{a_i} is *low* AND CP_{a_j} is *moderate OR moderate high* **THEN** CP_{Ch} is *greater than*

R11: **IF** CP_{a_i} is *low* AND CP_{a_j} is *high OR very high* **THEN** CP_{Ch} is *much greater than*

R12: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *near to zero* **THEN** CP_{Ch} is *much lower than*

R13: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *very low OR low* **THEN** CP_{Ch} is *lower than*

R14: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *moderate* **THEN** CP_{Ch} is *similar to*

R15: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *moderate high OR high* **THEN** CP_{Ch} is *greater than*

R16: **IF** CP_{a_i} is *moderate* AND CP_{a_j} is *very high* **THEN** CP_{Ch} is *much greater than*

R17: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *near to zero OR very low* **THEN** CP_{Ch} is *much lower than*

R18: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *low OR moderate* **THEN** CP_{Ch} is *lower than*

R19: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *moderate high* **THEN** CP_{Ch} is *similar to*

R20: **IF** CP_{a_i} is *moderate high* AND CP_{a_j} is *high OR very high* **THEN** CP_{Ch} is *greater than*

R21: **IF** CP_{a_i} is *high* AND CP_{a_j} is *near to zero OR very low* **THEN** CP_{Ch} is *much lower than*

R22: **IF** CP_{a_i} is *high* AND CP_{a_j} is *low OR moderate OR moderate high* **THEN** CP_{Ch} is *lower than*

R23: **IF** CP_{a_i} is *high* AND CP_{a_j} is *high* **THEN** CP_{Ch} is *similar to*

R24: **IF** CP_{a_i} is *high* AND CP_{a_j} is *very high* **THEN** CP_{Ch} is *greater than*

R25: **IF** CP_{a_i} is *very high* AND CP_{a_j} is *near to zero OR very low OR low* **THEN** CP_{Ch} is *much lower than*

R26: **IF** CP_{a_i} is *very high* AND CP_{a_j} is *moderate OR moderate high OR high* **THEN** CP_{Ch} is *lower than*

R27: **IF** CP_{a_i} is *very high* AND CP_{a_j} is *very high* **THEN** CP_{Ch} is *similar to*

Then, we compute the difference of the lowest acceleration with respect to the average of the rest of elements. We compute this difference by the

next SFP(*small*=$triangle_{MF}$(0, 0, 20), *medium*=$triangle_{MF}$(0, 20, 50), *high*=$trapezoid_{MF}$(20, 50, 100, 100)).

T_{Samin} : "The lowest acceleration took place between years {*previous year*}-{*current year*} by {*small* | *medium* | *high*} difference with respect to the average of the rest of years".

2 GLMP for Basic Reports

2.1 GLMP for analyzing the level of deforestation

1PM$_d$ = (U_d, y_d, g_d, T_d), where d stands for *deforestation* and:

U_d = (year, percentage of deforestation)

y_d = (A_d, W_{Ad}, R_{Ad}) where:

A_d = (*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*)

W_{Ad} = ($w_{Ad_{zero}}$, $w_{Ad_{near\ to\ zero}}$, $w_{Ad_{very\ low}}$, $w_{Ad_{low}}$, $w_{Ad_{moderate\ low}}$, $w_{Ad_{moderate}}$, $w_{Ad_{moderate\ high}}$, $w_{Ad_{high}}$, $w_{Ad_{very\ high}}$), with $w_{Ad_i} \in$ [0,1]

R_{Ad} = ($r_{Ad_{zero}}$ = 1, $r_{Ad_{near\ to\ zero}}$ = 1, $r_{Ad_{very\ low}}$ = 1, $r_{Ad_{low}}$ = 1, $r_{Ad_{moderate\ low}}$ = 1, $r_{Ad_{moderate}}$ = 1, $r_{Ad_{moderate\ high}}$ = 1, $r_{Ad_{high}}$ = 1, $r_{Ad_{very\ high}}$ = 1)

g_d :

g_{W_A}: SFP(*zero*=$triangle_{MF}$(0, 0, 0.001), *near to zero*=$triangle_{MF}$(0, 0.001, 0.1), *very low*=$triangle_{MF}$(0.001, 0.1, 15), *low*=$triangle_{MF}$(0.1, 15, 30), *moderate low*=$triangle_{MF}$(15, 30, 45), *moderate*=$triangle_{MF}$(30, 45, 55), *moderate high*=$trapezoid_{MF}$(45, 55, 60, 70), *high*=$trapezoid_{MF}$(60, 70, 80, 85), *very high*=$trapezoid_{MF}$(80, 85, 100, 100))

NOTE: SFP stands for Strong Fuzzy Partition. $triangle_{MF}$ is a membership function with triangular shape and three parameters (a, b, c) determining the vertices of the triangle. $trapezoid_{MF}$ is a membership function with trapezoidal shape and four parameters (a, b, c, d) determining the support (a-d) and the core (b-c) of the trapezoid.

T_d : "The deforestation in {*year*} was {*zero* | *near to zero* | *very low* | *low* | *moderate low* | *moderate* | *moderate high* | *high* | *very high*}".

2PM$_{Id}$ = (U_{Id}, y_{Id}, g_{Id}, T_{Id}), where Id means that this is an *Integrative* Perception Mapping related to *deforestation*:

U_{Id} = (CP_{d2000}, CP_{d2001}, CP_{d2002}, CP_{d2003}, CP_{d2004}, CP_{d2005}, CP_{d2006}, CP_{d2007}, CP_{d2008}, CP_{d2009}, CP_{d2010}, CP_{d2011}, CP_{d2012}, CP_{d2013}, CP_{d2014})

y_{Id} = (A_{Id}, W_{AId}, R_{AId}) where:

A_{Id} = ((Q_{T_1}, Q_{L_1}), ..., (Q_{T_1}, Q_{L_9}), (Q_{T_2}, Q_{L_1}), ..., (Q_{T_2}, Q_{L_9}), ..., (Q_{T_6}, Q_{L_1}), ..., (Q_{T_6}, Q_{L_9})), being Q_T=(*one*, *two*, *three*, *several*, *most*, *all*) and Q_L=(*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*)

W_{AId} = ($w_{AId_{11}}$ = (*one*, *zero*), ..., $w_{AId_{19}}$ = (*one*, *very high*), $w_{AId_{21}}$ = (*two*, *zero*), ..., $w_{AId_{29}}$ = (*two*, *very high*), ..., $w_{AId_{61}}$ = (*all*, *zero*), ..., $w_{AId_{69}}$ = (*all*, *very high*)), with $w_{AId_{ij}} \in$ [0,1]

R_{AId} = ($w_{AId_{11}} = 1$, ..., $w_{AId_{19}} = 1$, $w_{AId_{21}} = 1$, ..., $w_{AId_{29}} = 1$, ..., $w_{AId_{61}} = 1$, ..., $w_{AId_{69}} = 1$)

g_{Id} :

g_{W_A}: For each level of deforestation determined by the index j, i.e., A_d= (a_{d_1}, a_{d_2}, ..., a_{d_j}), we calculate the percentage of years contained at each $\alpha-$level ($N_{\alpha j}$), with $\alpha \in$ A= (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1), as follows:

$$N_{\alpha j} = \frac{1}{K} \sum_{k=1}^{K} F_\alpha(w_{d_j}[k]) \tag{4}$$

where $F_\alpha(z)$ is defined by (5), and $w_{d_j}[k]$ is the validity degree of CP_d, which is calculated for each year (k) being K the total number of evaluated years of each municipality.

$$F_\alpha(z) = \begin{cases} 1 & \text{if} \quad z \geq \alpha \\ 0 & \text{if} \quad z < \alpha \end{cases} \tag{5}$$

Then, we calculate the membership degree of each $N_{\alpha j}$ to each element in Q_T, e.g., $\mu_{Q_{T1}}(N_{\alpha j}) = one(N_{\alpha j})$. The shapes of these linguistic labels are determined by the total number of years K. The related SFP is as follows: (*one*=$trapezoid_{MF}$(0, 0, 0.1, 0.2), *two*=$triangle_{MF}$(0.1, 0.2, 0.3), *three*=$triangle_{MF}$(0.2, 0.3, 0.4), *several*=$triangle_{MF}$(0.3, 0.4, 0.6), *most*=$trapezoid_{MF}$(0.4, 0.6, 0.99, 1), *all*=$triangle_{MF}$(0.99, 1, 1)).

The last step is to calculate the average value of the membership degrees obtained for each $\alpha-$level using (6). The cardinality of the set A ($|\mathrm{A}| = 10$) represents the resolution degree.

$$w_{Id_{ij}} = \frac{1}{|\mathrm{A}|} \sum_{\forall \alpha \in \mathrm{A}} \mu_{Q_{T_i}}(N_{\alpha j}) \tag{6}$$

This final value contains the relevant information about the amount of years with (*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*) deforestation level.

T_{Id} :

If the combination of linguistic expressions in CP_{Id} with the greatest validity degree is $A_{Id_{61}}$ = (*all*, *zero*), then the generated sentence will be:

"Results show that in the last fifteen years there have not been deforestation".

Otherwise, we apply the following linguistic template: "Results show that {*the greatest* Q_T} the deforestation level was {*the greatest* Q_L}, and besides, {*the second greatest* Q_T} the deforestation level was {*the second greatest* Q_L}".

2.2 GLMP for analyzing the increment in the level of deforestation

1PM$_i$ $= (U_i, y_i, g_i, T_i)$, where i stands for *increment* and:

U_i = (the year, the increment in the percentage of deforestation)

$y_i = (A_i, W_{Ai}, R_{Ai})$ where:

A_i = (near to zero, very low, low, moderate, moderate high, high, very high)

$W_{Ai} = (w_{Ai_{near\ to\ zero}}, w_{Ai_{very\ low}}, w_{Ai_{low}}, w_{Ai_{moderate}}, w_{Ai_{moderate\ high}}, w_{Ai_{high}}, w_{Ai_{very\ high}})$, with $w_{Ai_i} \in [0, 1]$

$R_{Ai} = (r_{Ai_{near\ to\ zero}} = 1, r_{Ai_{very\ low}} = 1, r_{Ai_{low}} = 1, r_{Ai_{moderate}} = 1, r_{Ai_{moderate\ high}} = 1, r_{Ai_{high}} = 1, r_{Ai_{very\ high}} = 1)$

g_i :

g_{W_A}: SFP(*near to zero*=$triangle_{MF}(0, 0, 0.1)$, *very low*=$triangle_{MF}(0, 0.1, 15)$, *low*=$triangle_{MF}(0.1, 15, 30)$, *moderate*=$triangle_{MF}(15, 30, 45)$, *moderate high*=$triangle_{MF}(30, 45, 60)$, *high*=$trapezoid_{MF}(45, 60, 70, 80)$, *very high*=$trapezoid_{MF}(70, 80, 100, 100)$)

T_i : "The increment was {*near to zero* | *very low* | *low* | *moderate* | *moderate high* | *high* | *very high*} ({*the increment in the percentage of deforestation*}%)".

1PM$_{imax}$ $= (U_{imax}, y_{imax}, g_{imax}, T_{imax})$, where *imax* means *maximum increment* and:

U_{imax} = (year, deforestation increment)

$y_{imax} = (A_{imax}, W_{Aimax}, R_{Aimax})$ where:

$A_{imax} = (yes, no)$

$W_{Aimax} = (w_{Aimax_{yes}}, w_{Aimax_{no}})$, with $w_{Aimax_i} \in [0, 1]$

$R_{Aimax} = (r_{Aimax_{yes}} = 1, r_{Aimax_{no}} = 1)$

g_{imax} :

g_{W_A}: For each U_{imax}, we create a new CP_{imax} with $w_{Aimax_{yes}}$=1 and $w_{Aimax_{no}}$=0, in case of local maximum. Otherwise, $w_{Aimax_{yes}}$=0 and $w_{Aimax_{no}}$=1.

T_{imax} : "There {*was* | *was not*} a maximum". Notice that we select {*was*} when the validity degree associated to the linguistic expression *yes* is equal to 1 and {*was not*} in the other case.

$\mathbf{1PM}_{imin}$ = (U_{imin}, y_{imin}, g_{imin}, T_{imin}), where *imin* means *minimum increment* and:

U_{imin} = (year, deforestation increment)

y_{imin} = (A_{imin}, W_{Aimin}, R_{Aimin}) where:

A_{imin} = (*yes*, *no*)
W_{Aimin} = ($w_{Aimin_{yes}}$, $w_{Aimin_{no}}$), with $w_{Aimin_i} \in [0, 1]$
R_{Aimin} = ($r_{Aimin_{yes}} = 1$, $r_{Aimin_{no}} = 1$)

g_{imin} :

g_{W_A}: For each U_{imin}, we create a new CP_{imin} with $w_{Aimin_{yes}}$=1 and $w_{Aimin_{no}}$=0, in case of local minimum. Otherwise, $w_{Aimin_{yes}}$=0 and $w_{Aimin_{no}}$=1.

T_{imin} : "There {*was* | *was not*} a minimum". Notice that we select {*was*} when the validity degree associated to the linguistic expression *yes* is equal to 1 and {*was not*} in the other case.

2.3 GLMP for analyzing the acceleration in the deforestation rate

$\mathbf{1PM}_a$ = (U_a, y_a, g_a, T_a), where *a* stands for *acceleration* and:

U_a = (the previous year, the current year, the percentage of acceleration between these years)

y_a = (A_a, W_{Aa}, R_{Aa}) where:

A_a = (*zero*, *near to zero*, *very low*, *low*, *moderate low*, *moderate*, *moderate high*, *high*, *very high*)
W_{Aa} = ($w_{Aa_{zero}}$, $w_{Aa_{near\ to\ zero}}$, $w_{Aa_{very\ low}}$, $w_{Aa_{low}}$, $w_{Aa_{moderate\ low}}$, $w_{Aa_{moderate}}$, $w_{Aa_{moderate\ high}}$, $w_{Aa_{high}}$, $w_{Aa_{very\ high}}$), with $w_{Aa_i} \in [0, 1]$
R_{Aa} = ($r_{Aa_{zero}} = 1$, $r_{Aa_{near\ to\ zero}} = 1$, $r_{Aa_{very\ low}} = 1$, $r_{Aa_{low}} = 1$, $r_{Aa_{moderate\ low}} = 1$, $r_{Aa_{moderate}} = 1$, $r_{Aa_{moderate\ high}} = 1$, $r_{Aa_{high}} = 1$, $r_{Aa_{very\ high}} = 1$)

g_a :

g_{W_A}: SFP(*zero*=$triangle_{MF}$(0, 0, 0.001), *near to zero*=$triangle_{MF}$(0, 0.001, 0.1), *very low*=$triangle_{MF}$(0.001, 0.1, 15), *low*=$triangle_{MF}$(0.1, 15, 30), *moderate low*=$triangle_{MF}$(15, 30, 45), *moderate*=$triangle_{MF}$(30, 45, 55), *moderate high*=$trapezoid_{MF}$(45, 55, 60, 70), *high*=$trapezoid_{MF}$(60, 70, 80, 85), *very high*=$trapezoid_{MF}$(80, 85, 100, 100)).

T_a : "The acceleration was {*zero* | *near to zero* | *very low* | *low* | *moderate low* | *moderate* | *moderate high* | *high* | *very high*} and *negative* | *positive* | *zero* ({*the percentage of acceleration*}%)".